INVESTIGATION OF GLASS-METAL COMPOSITE MATERIALS

P. A. Lockwood

Contract NOrd 15764

Fifteenth Quarterly Progress Report

Covering Period September 15, 1959 to December 15, 1959

Owens-Corning FIBERGLAS Corporation
Basic and Applied Research Center
Newark, Ohio

TABLE OF CONTENTS

INVESTIGATION OF GLASS-METAL COMPOSITE MATERIALS

INTRODUCTION

In the period September 15, 1959, to December 15, 1959, research was reactivated directed to the development of high temperature composites. Effort for the quarter comprised forming high temperature fibers; studies of tensile strengths of single fibers both bare and aluminum coated; and forming crystalline fibers for study of their physical properties and for compositing with metals. Also during the period experiments begun during the preceding quarter concerning the basic behavior of glass-aluminum composites were completed. An experiment planned in the Fourteenth Quarter was initiated concerning properties in tension of glass-aluminum composites in which the reinforcement was high temperature fibers.

The work reported represents the combined efforts of Messrs. M. Chrisman, W. Edmunds, B. Garick, R. Harris, N. Leedy, P. Lockwood, E. Mattern, G. Wince, and Dr. G. R. Machlan and other members of the Basic and Applied Research Laboratories of the Owens-Corning Fiberglas Corporation. Physical property measurements of glass reinforced metal test bars were performed at The Ohio State Engineering Experiment Station of The Ohio State University in Columbus, Ohio, under the direction of Dr. T. S. Shevlin and at the Owens-Corning Fiberglas Testing Laboratories.

SUMMARY

The direction of work during this quarter has been varied from the work reported in previous quarters due to renewed interest in high temperature composites. This change was discussed in the Fourteenth Quarterly Progress Report. Experiments with high temperature composite component materials were started at the beginning of the Fifteenth Quarter.

The Owens-Corning Fiberglas Corporation has been studying high temperature glass fibers with considerable success and it was well known that compositions in the SiO_2-Al_2O_3-MgO ternary, the SiO_2-Al_2O_3-CaO ternary, and the SiO_2-Al_2O_3-MgO-CaO quaternary systems offered a fertile field for fibers with exceptional high temperature strengths. Relying on glass forming knowledge gained in earlier studies, four glasses from the CaO ternary system were studied along with two glasses from the quaternary system and four glasses from the MgO ternary system. While most of the glasses studied showed an improvement in tensile strength both bare and aluminum coated compared to "E" glass, X-994 in the MgO ternary system shows the most promise.

The X-994 glass was formed at over 3100°F and had an average tensile strength in the uncoated condition of 635,000 psi and a tensile strength of 178,000 psi coated with aluminum, roughly 50,000 psi greater than the tensile strength of aluminum-coated "E" glass fibers. Other glasses from the three composition fields will be examined as they become available.

Originally it was thought that aluminum coatings applied to candidate high temperature fibers at forming would serve to eliminate those glasses particularly susceptible to damage by reactive metals such as zirconium

and titanium. Also, the coating step was thought to be a means for indicating those glasses which might lose strength rapidly at or above 1200°F. It was found, however, that even the more refractory fibers were not immune from attack by molten aluminum and were being unduly penalized. Future screening will be done by testing bare fibers at room temperature, 1000, 1500, 1800, and 2000°F. Aluminum coated fibers will be used only when information useful to the high temperature composite program may result.

A previously outlined experiment was initiated with the object of determining if aluminum coated high temperature fibers showing higher strengths than "E" glass aluminum-coated fibers would yield stronger glass-aluminum composites. Vacuum injection cast and hot pressed composites were made embodying one glass from each of the three composition fields. Tensile strengths of the composites were measured at room temperature, 500, 700, and 1000°F. The results did not differ significantly from those obtained previously for "E" glass aluminum composites.

Microscope studies were started of high temperature crystalline fibers formed by a process known from Owens-Corning Fiberglas research. A small supply of these fibers has been backlogged.

Strong fairly coherent green composites of crystalline fibers and -325 mesh stainless steel powder were made by hot pressing at 1000°F and 25 tons per square inch. Sintering was attempted with some success by heating pieces of the green compact for several hours at 1800 - 2000°F while embedded in a protective mixture of powdered alumina and graphite. A considerable degree of sintering occurred with only minor

oxidation from air entrapped in the sample. The results, while merely preliminary, indicate that fabrication of composites by these or similar methods is possible. Several methods of making green compacts are under study and equipment is being designed for trying them.

During the quarter Dr. Edward Saibel, consultant from Rensselaer Polytechnic Institute, supplied a paper on "The Poisson's Ratio of Composite Materials." In anticipation of this paper, "E" glass-aluminum composites suitable for testing in compression were made in the Fourteenth Quarter. During the Fifteenth Quarter, The Ohio State University determined Poisson's Ratio for these bars with the load applied in several different directions in relation to fiber orientation. Consonant with assumptions adopted in the theoretical treatment, Dr. Saibel predicted that the bars would behave as homogeneous common structural units and that Poisson's Ratio for the composites would fall between those for the components, the value depending on total cross section of each component. The results did not follow the predictions. Poisson's Ratio for the composites fell completely outside the values for either of the two components taken separately. Extreme variation was noted between Poisson's Ratio in tension and compression.

DISCUSSION

High Temperature Composites

At the end of the Fourteenth Quarter the Navy Bureau of Ordnance indicated that their interest in high temperature composite materials took precedence over previous work in developing theoretical concepts to explain the behavior of composite materials. Recent developments in the field of high temperature glass fibers and high temperature crystalline fibers gave promise that re-entry into the field of high temperature composites might be more fruitful than when first attempted in early 1958. A re-evaluation of the program was carried on during the month of September, 1959. A review of fabrication techniques, especially recent developments, indicated that there were several ways in which fiber-metal composites of high temperature interest might be made on a laboratory scale without large capital outlays.

The initial fiber screening procedure consisted of forming fibers from various high temperature glasses previously investigated and testing them for tensile strength. It was decided that a good criterion for high temperature properties of glasses would be to aluminum coat the fibers at forming and test them. Those fibers which had good room temperature strength bare and aluminum coated would then be tested at various high temperatures - 1000°F, 1500°F, 1800°F, and 2000°F. The aluminum coating was calculated to eliminate those glasses particularly susceptible to reactive metals and also to indicate those glasses which might lose strength rapidly at or above 1200°F.

From previous Owens-Corning Fiberglas experiences in the high temperature glass forming fields, it was concluded that the SiO_2-Al_2O_3-MgO ternary, SiO_2-Al_2O_3-CaO ternary, and the SiO_2-Al_2O_3-MgO-CaO quaternary offered the three most fruitful composition fields for the investigation of high temperature glasses likely to have adequate resistance to reactive metals. Table I gives the results for ten glasses that were tested during this quarter. To check the new forming system a control set of "E" glass fibers were tested. It will be noted that Glass X-994 from the silica-alumina-magnesia ternary field showed the most promising results with an average of 635,000 psi in the as-pulled condition and 178,000 psi aluminum coated. Tests of X-994 fibers formed at higher temperatures and reruns of X-994 and X-995 have been delayed by forming position difficulties.

A re-evaluation of the results from testing of fibers, both bare and aluminum coated, has led to the conclusion that aluminum is so highly reactive that even the more refractory fibers are not immune from attack. The fibers are therefore being unduly penalized. For this reason the majority of fibers in the future will be tested in the bare uncoated condition at room temperature, 1000°F, 1500°F, 1800°F, and 2000°F whenever possible.

Concurrent with the search for suitable high temperature glass fibers, a series of hot pressed and vacuum injection cast aluminum composite bars were made incorporating promising glasses from each of the three composition fields. The object of the experiment was to assess the effect of the higher strength aluminum-coated fibers on tensile

TABLE I

TENSILE TESTS OF HIGH TEMPERATURE FIBERS

Glass System Glass Number		Average Tensile Strength - Psi	
		As Pulled	Aluminum Coated
SiO_2-Al_2O_3-MgO			
X-30-A	1st Trial*	494,000	158,000
	2nd Trial	503,000	156,000
X-35-A		The Glass Devitrified Giving Widely Varying Results	
X-994	1st Trial	635,000	178,000
	2nd Trial	Bushing Troubles	
X-995	1st Trial	Bushing Troubles	
SiO_2-Al_2O_3-CaO			
X-870	1st Trial	447,000	132,000
	2nd Trial	492,000	139,000
X-871		This Glass Would Not Produce a Fiber of Uniform Diameter	
X-969	1st Trial	448,000	136,000
X-894-A	1st Trial	291,000	131,000
SiO_2-Al_2O_3-CaO-MgO			
X-37-B	1st Trial	469,000	128,000
X-673	1st Trial	473,000	110,000
Control			
"E"	1st Trial	410,000	128,000

*Thirty-six or more fiber breaks were averaged in each trial.

strength of the composites as compared to "E" glass reinforcement. Fibers were oriented parallel in both types of bars, the vacuum injection cast set containing 20 per cent by volume fibers and the hot pressed set 50 per cent by volume. The glasses chosen were X-870 (lime ternary), X-30-A (magnesia ternary), and X-37-B (lime-magnesia quaternary). With the exception of vacuum castings of X-37-B for which results from previous work were available, the bars were tested and the results are shown in Tables II and III.

The test results did not show the expected improvement in tensile strength over "E" glass-aluminum composites. The reason for this might be that the strength increase observed for the aluminum-coated high temperature fibers was not large enough to show in the overall average composite strengths or that the fiber strength is not a major influence on composite strength. Another and more likely possibility is that the fiber strength was further degraded during the compositing operation until there was no difference in strength between "E" glass fibers and the high temperature fibers.

Also of great interest are crystalline fibers which have been developed by the Owens-Corning Fiberglas Pioneering Research Laboratories. These fibers show great promise as high temperature material. Forming of these fibers was started on a scale patterned after the process established by the Pioneering Research group and is ahead of schedule.

Trials have been started to incorporate the crystalline fibers into high temperature composites. While the first trials were very crude and were attempted only to see how the materials would behave, the result of

TABLE II

ULTIMATE TENSILE STRENGTHS OF COMPOSITES
CONTAINING HIGH TEMPERATURE GLASS FIBERS

Vacuum Injection
20% by Volume Glass
Orientation - Parallel
Coating - 1100 Aluminum
Matrix - 1100 Aluminum

Test Temp. °F	X-870+	X-30-A+	X-37-B**	"E"*	"E"**
Room	15,360 psi	14,816 psi	32,700 psi	14,000 psi	
	16,640	14,024	32,200	13,700	
	14,720	9,891	31,750	17,500	
	13,800	11,212	24,600	13,780	
	13,120	11,366	27,700	13,780	
Average	14,728	12,262	29,890	14,500	30,620 psi
500°F	11,379 psi	12,525 psi		12,000 psi	
	12,512	12,136		18,000	
	12,012	9,273		14,500	
	9,642	11,803			
	11,089	11,017			
Average	11,326	11,351		14,930	27,250 psi
700°F	14,064 psi	8,758 psi	15,150 psi	15,250 psi	
	17,257	11,512	14,200	13,650	
	17,585	8,000	9,190	13,620	
	15,365	9,014	12,360	12,900	
	11,831	8,375	11,180		
Average	15,220	9,132	12,520	13,855	27,250 psi
1000°F	10,721 psi	4,460 psi	6,140 psi	7,600 psi	
	7,464	5,611	6,170	12,666	
	10,398	2,970	7,200	7,930	
	10,660	5,240	3,860	8,225	
	9,840	7,129		10,400	
Average	9,817	5,082	5,835	9,360	15,525 psi

*Previously reported data.
**Previously reported data. 2014 Aluminum matrix solution and aged.
+X-870 bars showed a few small voids in the fracture.
X-30-A bars showed many small voids and several large voids in the fractures.

TABLE III

ULTIMATE TENSILE STRENGTHS OF COMPOSITES
CONTAINING HIGH TEMPERATURE GLASS FIBERS

Hot Pressed
50% by Volume Glass
Orientation - Parallel
50% by Volume 1100 Aluminum

	X-870	X-30-A	X-37-B	"E"
	16,311 psi	35,622 psi	26,804 psi	29,300 psi
	25,821	30,526	31,347	26,900
	20,871	35,118	23,529	18,800
	23,000	34,894	32,721	16,700
	23,110	35,823	31,437	22,300
	29,167	22,331	30,585	37,500
	26,288	23,085	28,378	22,950
	23,573	22,819	25,579	
	18,996	23,490	26,231	
	32,150	35,773	24,190	
	23,092	36,419	28,256	
	31,967	32,500	30,931	
	18,938	24,329		
	20,291	21,408		
	15,292	36,327		
	18,725	20,122		
	15,400	24,472		
	19,679			
	13,556			
	11,515			
High	32,150 psi	36,469 psi	31,437 psi	37,500 psi
Average	21,871	29,418	28,332	24,921
Low	11,515	20,122	23,529	18,800

Some of the above bars were supposed to be tested at elevated temperatures but an error in the Testing Division resulted in all bars being tested at room temperature.

the fabricating process was deemed satisfactory. Twenty weight per cent crystalline fibers were mixed with eighty weight per cent 302 Stainless Steel and a glycol binder. This mixture was placed in a die which was heated to 1000°F and pressed at approximately 25 tons per square inch. The green compact which resulted from the hot pressing operation showed sufficient strength to allow for all normal handling of the sample such as removing it from the die and for normal inspection procedures including cutting the sample and making metallographic examinations. While the sample withstood all these handling operations, it was found that the standard techniques for polishing and etching did not succeed. The net result was that whereas the sample could be investigated under the microscope, photographic examinations were not too successful. Investigations will be made into improving metallographic techniques for future samples.

Sections were taken from the first two samples made for trial sintering operations to see what could be done without an atmosphere furnace. An atmosphere furnace is now on order but delivery is scheduled for late in the Sixteenth Quarter. Because data on composite properties are needed earlier, it is necessary that sintering be started before the furnace is available. The first trial was aimed at seeing how these sintering experiments could be made. It was decided that the sample could be placed in a crucible with a mixture of aluminum oxide mixed with about ten weight per cent graphite powder. The graphite should act as a reducing agent by reacting with the oxygen of entrapped air, and the aluminum oxide should prevent circulation of air near the sample. This system works quite well except that absorbed gases in the green compact contribute to a slight

amount of oxidation during the sintering operation. The first trial was at 1800°F for 2 1/2 hours and the second trial was at 2000°F for 2 hours. A considerable amount of sintering did take place although it was in no way deemed that complete densification had occurred. The results of the experiments, though, were quite satisfactory especially in view of the fact that it was just an attempt to see if this method would suffice for preliminary tests.

Investigations Into the Basic Nature of Composites

As the Fifteenth Quarter represents a change-over time in emphasis on the research conducted, certain experiments from the Fourteenth Quarter had not been completed. The first of these experiments was an investigation into a change in the hot pressing technique which had been developed during the first year of the contract. This change has been mentioned previously, but the variables involved had not been studied. The major change was the use of a small die which could be heated uniformly in a furnace and then placed in a press to form the bars. Earlier dies were heated by imbedded electric resistance elements and suffered from large point-to-point temperature variations. The preheat temperature, preheat time, and time of pressing had to be studied for their effects on the strength of the composite. Table IV shows the initial trials in which a general traverse of the described variables was made and visual appearance of the bars was used as a criterion for changing the variables. After the better bars were tested (see Table IV), a second set of bars was prescribed from a statistical study of the first group. The results for this

TABLE IV

EFFECT OF FORMING VARIABLES ON TENSILE STRENGTH OF HOT PRESSED GLASS-ALUMINUM COMPOSITES

Preheat Temp.	Preheat Time	Press Time 8.0 Tsi	Press Time 31.5 Tsi	Thickness Inches	Tensile Str.-Psi	Appearance
670°F	30 min.	10 min.	5 min.	N.G.	—	Poor
670	45	10	5	N.G.	—	Poor
750	60	10	5	N.G.	—	Poor
800	45	10	5	N.G.	—	Poor
900	60	10	5	.186	5,889	Good
1000	60	10	5	.189	6,065	Good
1000	45	5	5	.185	8,880	Good
1000	45	1	1	.192	2,772	Fairly Good
1000	60	5	1	.188	9,881	Fairly Good
1000	60	5	5	.186	14,177	Fairly Good
1000	60	5	5	N.G.	—	N.G.
1000	110	5	–	.186	15,049	Good
1000	100	3	–	.182	22,026	Good
1000	60	1	–	.185	15,168	Good
1000	60	1	–	.188	15,648	Good
1000	60	1	–	.188	10,924	Good
900	60	1	–	.188	6,596	Good
900	60	1	–	.188	10,638	Good
850	60	1	–	.188	2,554	Good
850	60	1	–	.186	3,700	Good
850	60	1	–	.188	9,342	Good
750	60	1	–	.191	3,542	Good
750	60	1	–	.200	1,597	Fair
750	60	1	–	.201	5,876	Fair
750	60	1	–	.204	1,754	Fair
800	60	1	–	.201	4,374	Good

second group of bars (shown in Table V) indicate that several combinations of preheat time, temperature, and time of pressing may be used and that probably the most significant factor in composite strength is the degree of perfection of the aluminum coating on the fibers rather than the hot pressing process variables.

On October 8, 1959, Dr. Edward Saibel, a consultant from Rensselaer Polytechnic Institute, presented his third paper on properties of composite materials. Since Dr. Saibel had indicated the area in which he would be working, advance samples were prepared for determination of Poisson's Ratio in compression at The Ohio State University. The samples consisted of twenty per cent by volume "E" glass fibers parallel oriented vacuum injection cast with 2014 aluminum. The bars were heat treated to the solution and aged condition. Using the assumption that the bars act as homogeneous units when subjected to a load and that there is no relative slip between the fibers and the surrounding metal, Dr. Saibel calculated that the observed Poisson's Ratio for the composite bars should lie between the Poisson's Ratio of the glass and that of the aluminum in direct proportion to the total cross section of each.

Figures 1, 2, and 3 present the experimental data for compressive loads placed on cubes of the composite material with the fibers oriented in different directions to the load. The Poisson's Ratio was calculated from the slopes of the major sections of the curves. Surprisingly the Poisson's Ratio so determined was considerably higher than predicted and in the opposite direction to that found when determined in tension. An earlier determination of Poisson's Ratio for 2014 aluminum-"E" glass

TABLE V

EFFECT OF FORMING VARIABLES ON TENSILE STRENGTH
OF HOT PRESSED GLASS--ALUMINUM COMPOSITES

Preheat Temp.	Preheat Time	Press Time 8.0 Tsi	Thickness Inches	Tensile Str.-Psi	Appearance
900°F	64 min.	1 min.	.221	42,005	Good App. Slight Warp
900	64	1	.222	24,549	"
900	64	1	.224	25,382	"
950	81	1	.227	23,651	"
950	81	1	.222	25,090	"
950	81	1	.226	23,894	"
900	64	3	.221	24,537	"
900	64	3	.225	21,517	"
900	64	3	.226	28,139	"
950	81	3	.224	22,585	"
950	81	3	.222	25,385	"
950	81	3	.222	24,244	"
900	81	3	.222	37,377	"
900	81	3	.219	21,577	"
900	81	3	.225	19,824	Good No Warping
950	64	3	.234	15,625	"
950	64	3	.227	21,038	"
950	64	3	.220	25,430	"
950	64	1	.224	25,670	"
950	64	1	.221	26,582	"
950	64	1	.230	21,087	"
900	81	1	.222	13,000	"
900	81	1	.223	19,520	"
900	81	1	.220	25,800	"
925	72	1.42	.222	34,144	"
925	72	1.42	.224	24,374	"
925	72	1.42	.223	31,790	"
1000	60	3	.219	27,178	"
1000	60	3	.219	30,571	"
1000	60	3	.220	23,741	"

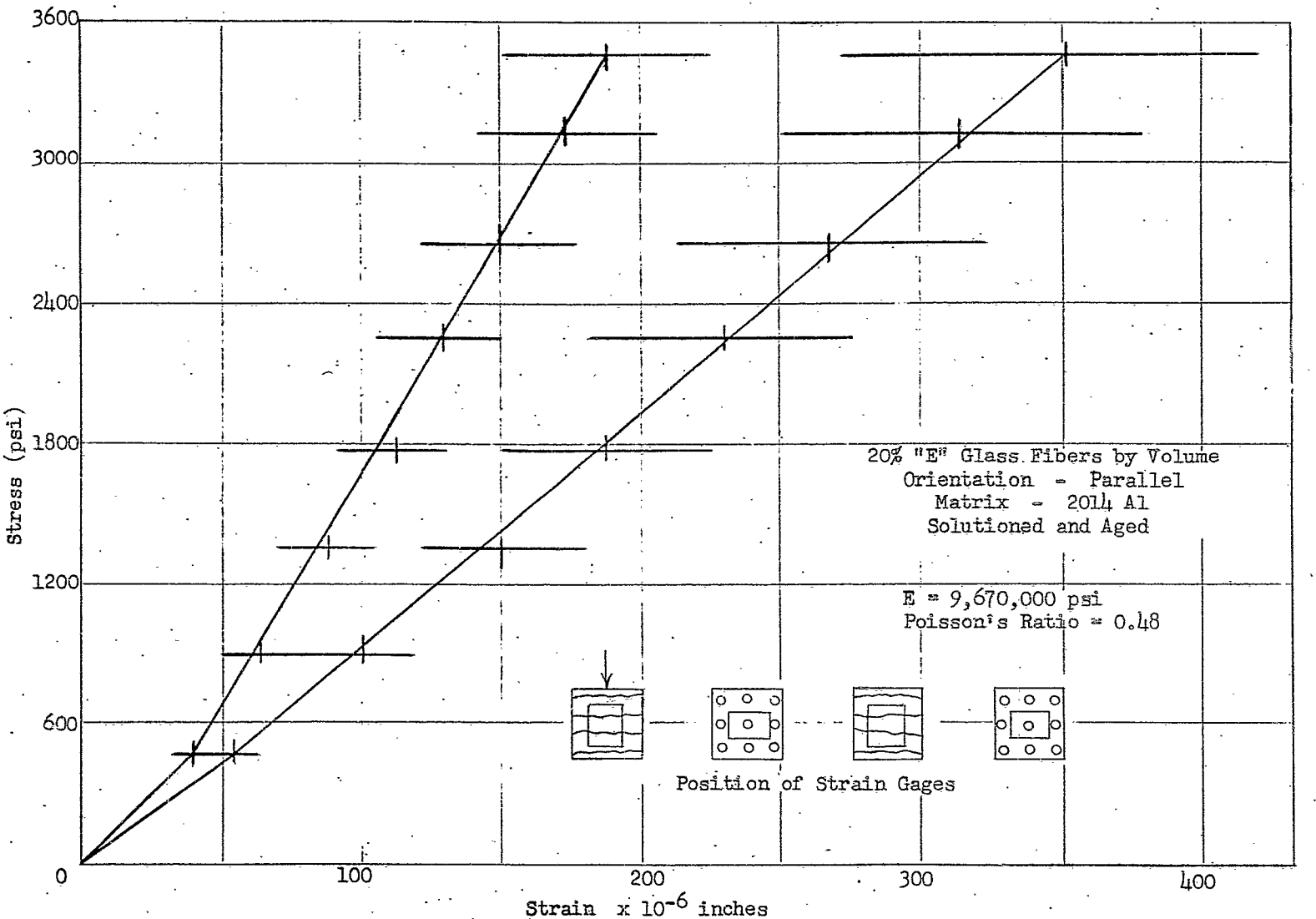

Figure 1 - Poisson's Ratio in Compression of Aluminum Composites

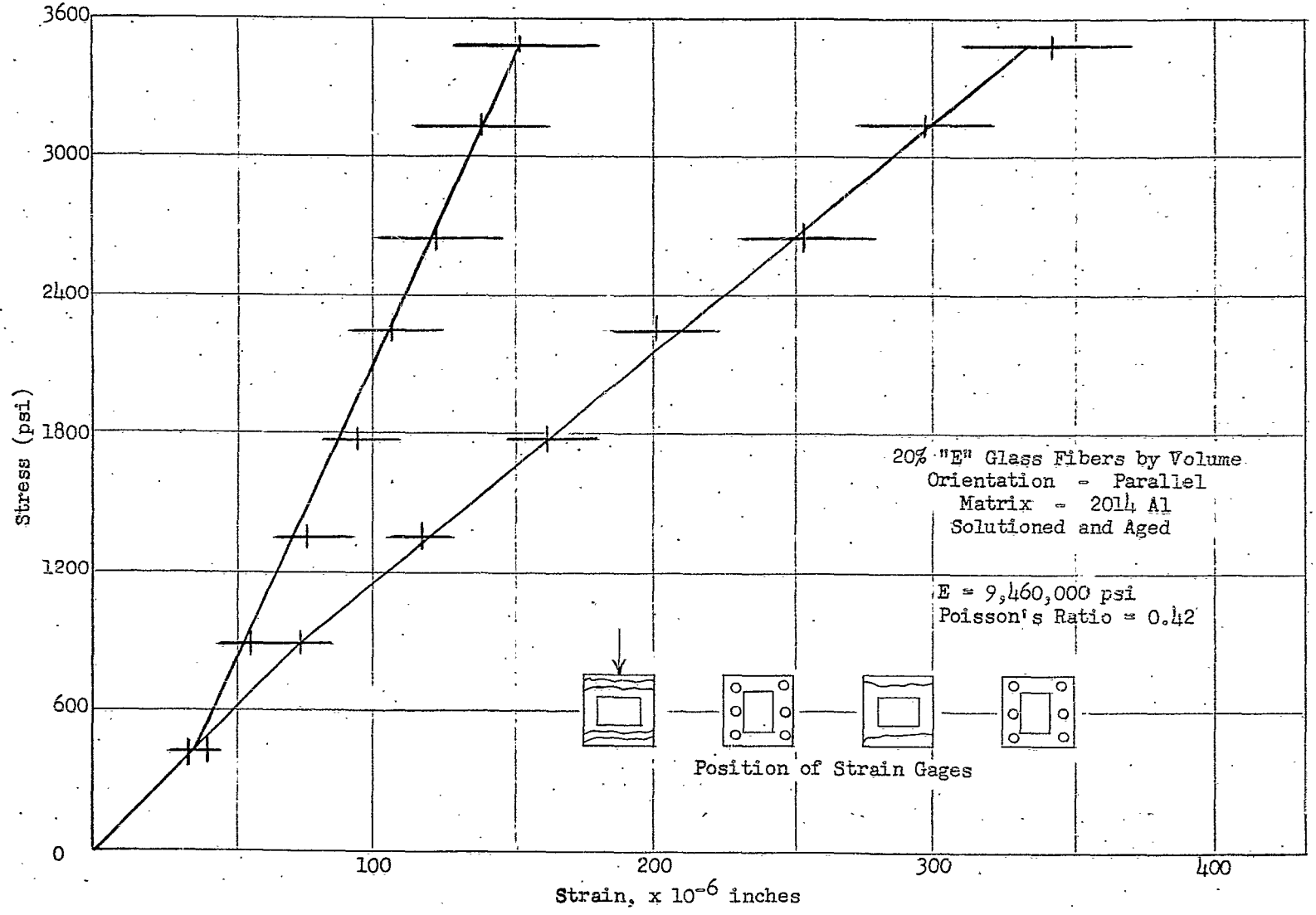

Figure 2 - Poisson's Ratio in Compression of Aluminum Composites

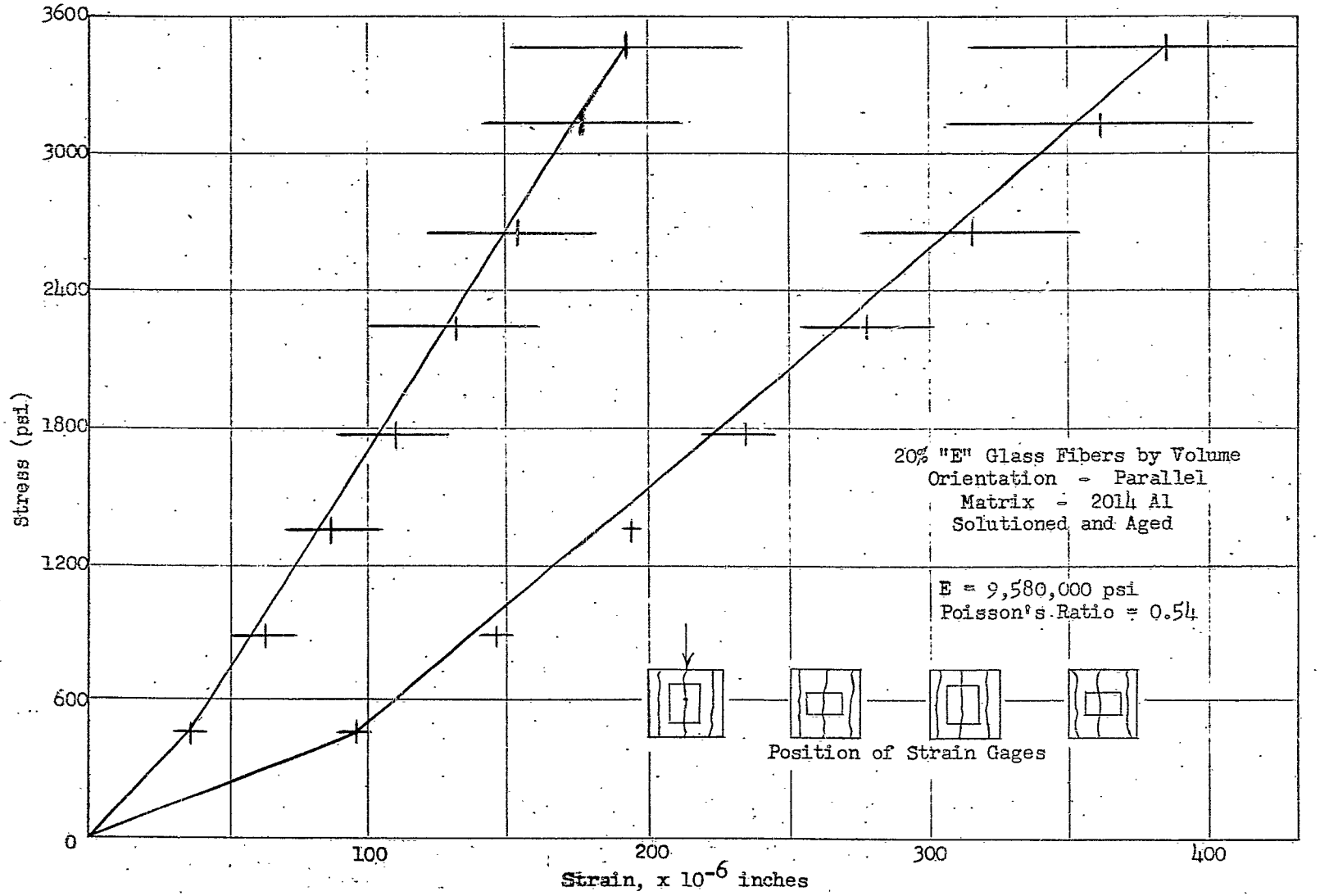

Figure 3 - Poisson's Ratio in Compression of Aluminum Composites

composites in tension yielded an average value of 0.100. An assumption from these results would be that the bars do not act as homogeneous units when subjected to load and that there must be some relative slip between the glass fibers and the surrounding metal. Comparative results from composites made of "E" glass fibers parallel oriented in 2014 aluminum matrixes in the T-6 condition certainly indicate that there is a gross difference between the reaction of the composite material when loads are applied in tension versus compression (see Table VI).

TABLE VI

COMPARISON OF PHYSICAL PROPERTIES
OF GLASS-ALUMINUM COMPOSITES
IN TENSION AND COMPRESSION

20% by Volume Parallel Oriented "E" Glass
2014 Aluminum Matrix
Solution and Aged

	Tension	Compression
Average Ultimate Strength	27,500 Psi	125,000 Psi
Ultimate Elongation	.7%	10.0 %
Modulus of Elasticity	10,200,000 Psi	10,000,000 Psi
Poisson's Ratio	.100	.500

FUTURE PLANS

The Owens-Corning Fiberglas Research Laboratory will be moved from its present location in the Newark Plant to a new location in the Owens-Corning Fiberglas Technical Center at Granville, Ohio. It is expected that this will cause some delay in the research effort but planning is such that the delay will be kept to an absolute minimum.

Three systems of fabricating composites will be studied as the equipment for them becomes available. The first method will be hot pressing which is a modified powder metallurgical technique. The second method is slip casting which has been recently developed for metals but is an old system for forming ceramic materials. The third method is low temperature, low pressure extrusion which again is not a metallurgical technique but a ceramic technique. Delivery of an atmosphere furnace is expected to take some time and attempts will be made to produce a small atmosphere furnace which can be used to do preliminary testing and to investigate techniques for sintering without a closely controlled atmosphere such as was mentioned in the "Discussion" section of this report. A considerable amount of study will have to be made on sintering of green composites, as this was one of the areas of difficulty during the experiments in 1958 which has not been modified by new techniques.

New high temperature glass fibers will be examined as they become available by making tensile strength tests at room temperature, 1000°F, 1500°F, 1800°F, and 2000°F.

A P P E N D I X

PAPER BY DR. EDWARD SAIBEL

"The Poisson's Ratio
of Composite Materials"

On Poisson's Ratio of Composite Materials

by

E. Saibel

I. Tadjbakhsh

October 8, 1959

Introduction

In the following a method is described for determination of the Poisson's ratio of composite glass-reinforced bars. It is assumed that such bars act as a unit when subjected to load and that there is no relative slip between the glass fibers and the surrounding material. The two cases of uniform and non-uniform fiber distribution have been discussed separately. Expressions have been obtained for the "observed" Poisson's ratio of the composite bar when the applied load is in the direction of glass fibers.

Nomenclature

x, y, z	=	Cartesian co-ordinate axes.
f	=	area of glass fibers per square inch.
ν_1	=	Poisson's ratio of glass.
ν_2	=	Poisson's ratio of metal.
ν	=	observed Poisson's ratio of composite bar.
P	=	applied load in tension or compression.
e_x, e_y, e_z	=	components of strain
$\sigma_x, \sigma_y, \sigma_z$	=	components of stress
E	=	modulus of elasticity

Case I. Uniform Fiber Distribution

Referring to figure 1, we consider a composite cube 1" x 1" x 1" in which the fibers are oriented in the direction of the x-axis.

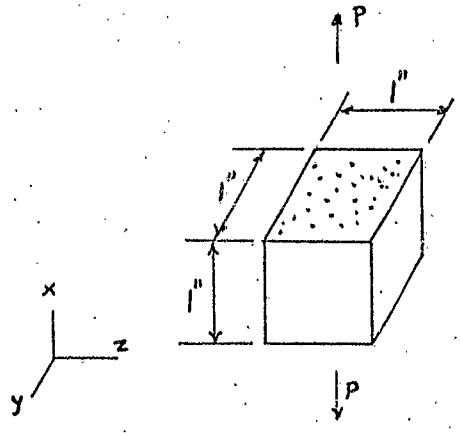

Fig. 1

Now, a body of original volume Vo when subjected to pure homogeneous extension or compression undergoes a change of volume ΔV_o given by

$$\Delta V_o = V_o \left[(1+e_x)(1-\nu e_x)(1-\nu e_x) - 1 \right] \tag{1}$$

Retaining only the linear terms in the above expression we get

$$\Delta V_o = V_o \, e_x (1-2\nu) \tag{2}$$

This same result can of course be obtained also from consideration of Hooke's Law which in the case of uniaxial loading states

$$\begin{cases} e_x = \dfrac{\sigma_x}{E} \\ e_y = -\nu \dfrac{\sigma_x}{E} \\ e_z = -\nu \dfrac{\sigma_x}{E} \end{cases} \tag{3}$$

Adding these equations we obtain the change in a unit volume as

$$\Delta V_0 = e_x + e_y + e_z = e_x (1-2\nu)$$

This result being established, the change in the volume of glass fibers of the cube considered previously is given by

$$\Delta V_1 = f\, e_x (1-2\nu_1) \tag{4}$$

and the metal by

$$\Delta V_2 = (1-f)\, e_x (1-2\nu_2) \tag{5}$$

The observed change in volume of composite bar is given by

$$\Delta V = e_x (1-2\nu) \tag{6}$$

Obviously we must have

$$\Delta V = \Delta V_1 + \Delta V_2 \tag{7}$$

which after substitution and simplification yields

$$\nu = \nu_2 + f(\nu_1 - \nu_2) \tag{8}$$

It is noted that since $o \leq f \leq 1$ and $o \leq \nu_1 , \nu_2 \leq \frac{1}{2}$, the observed Poisson's ratio always falls in the range

$$o \leq \nu \leq \frac{1}{2} \tag{9}$$

Case II. Non-Uniform Fiber Distribution

Referring to figure 2 we consider a body A under pure extension or compression in direction z and let f (x, y) designate the fiber-area distribution function.

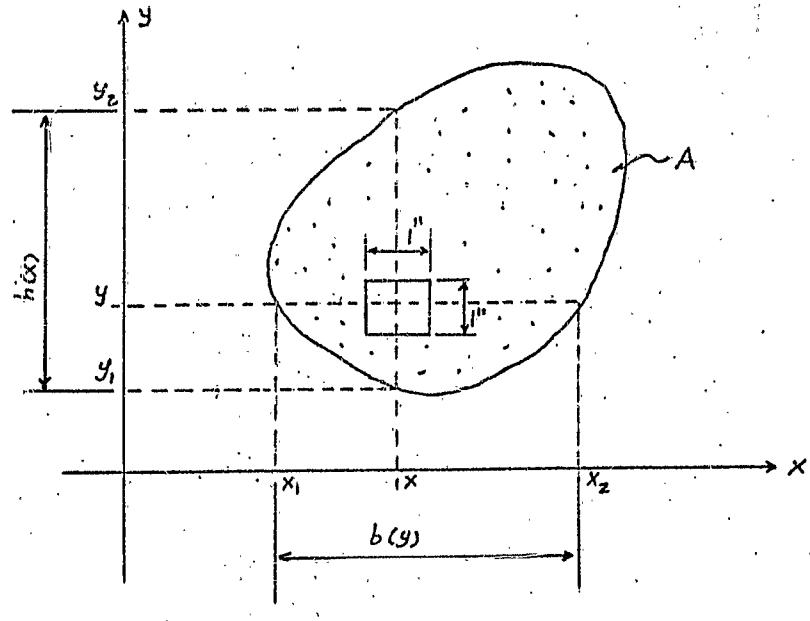

Fig. 2

Then for an element of A located at the point (x,y) we can write in the small

$$\nu = \nu_2 + f(x,y)(\nu_1 - \nu_2) \qquad (10)$$

Due to a strain e_z this element under goes lateral strains equal to νe_z. The total lateral contraction or elongation from y_1 to y_2 is given by

$$dh = \int_{y_1}^{y_2} \nu e_z \, dy = e_z \int_{y_1}^{y_2} \nu \, dy \qquad (11)$$

The observed strain e_y is then equal to

$$e_y = \frac{dh}{h} = \frac{e_z}{h(x)} \int_{y_1}^{y_2} \nu \, dy \qquad (12)$$

and the observed Poisson's ratio in direction y

$$\nu_y = \nu_y(x) = \frac{e_y}{e_z} = \frac{1}{h(x)} \int_{y_1}^{y_2} \nu \, dy \qquad (13)$$

Similarly we have in direction x

$$\nu_x = \nu_x(y) = \frac{1}{b(y)} \int_{x_1}^{x_2} \nu \, dx$$

where ν is given by equation (9). It is to be observed again that since $0 \leq \nu \leq \frac{1}{2}$ both quantities ν_x and ν_y fall in the range

$$0 \leq \nu_x, \nu_y \leq \frac{1}{2} \qquad (15)$$

for all x and y.